2D Poisson problem

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1 Problem Statement

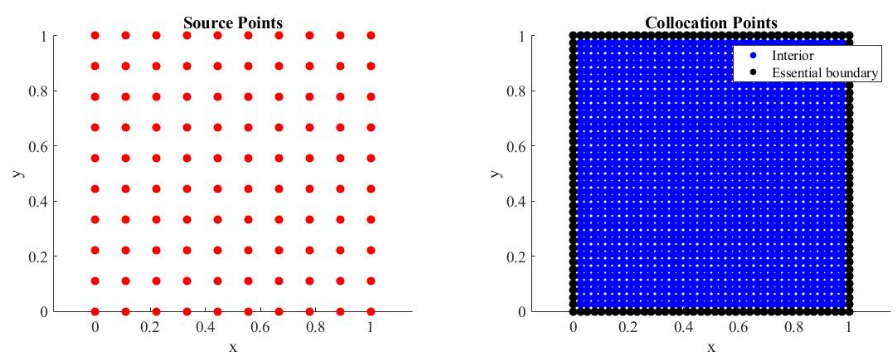
Consider the 2D Poisson problem with essential boundary conditions as given below:

$$\begin{aligned} \Delta u(x,y) &= (x^2+y^2)e^{xy} \qquad && \text{in} \quad \Omega = \{(x,y)|0<x<1, 0<y<1\} \\ u(x,y) &= e^{xy} && \text{on } \partial\Omega = \{(x,y)|x=0,1 \text{ and } y=0,1\} \end{aligned}$$

The analytical solution to the problem is $u(x,y) = e^{xy}$.

2 Numerical Analysis

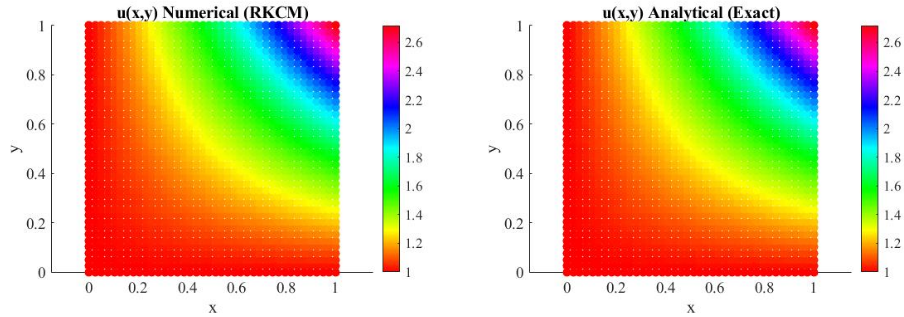
The domain is discretized using 10×10 source points and 40×40 collocation points which do not overlap as shown in Figure 1.



(a) Source points (b) Collocation points

Figure 1: Domain discretization

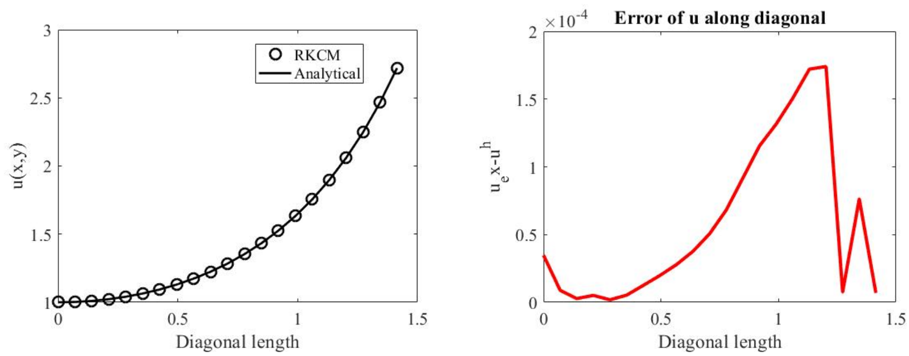
The weight on the essential boundary is taken to be $\sqrt{\alpha^g} = N_S$, where N_S is the number of source points. The numerical and analytical solutions in the 2D domain are compared in Figure 2.



(a) Numerical (b) Analytical

Figure 2: Comparison of numerical and analytical solution in 2D domain

Additionally, the numerical solution using RKCM obtained along the diagonal line passing through the points $(0, 0)$ and $(1, 1)$ is plotted in Figure 3 and compared with the analytical solution. The error between the two solutions along this diagonal line is also plotted.



(a) Solution comparison (b) Error

Figure 3: Comparison of numerical and analytical solution along the diagonal line

It can be seen that the numerical RKCM result is very close to the analytical solution.